Physics: Mechanics
Subject code: BSC-PHY-104G

CE

II^st Semester

Unit 2: Mechanics of Particles in Motion and Harmonic Motion

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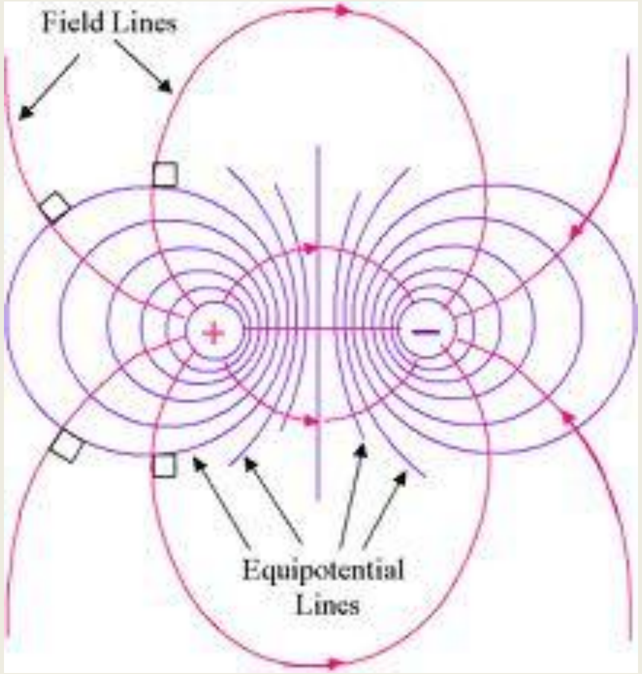
Equipotential Surfaces

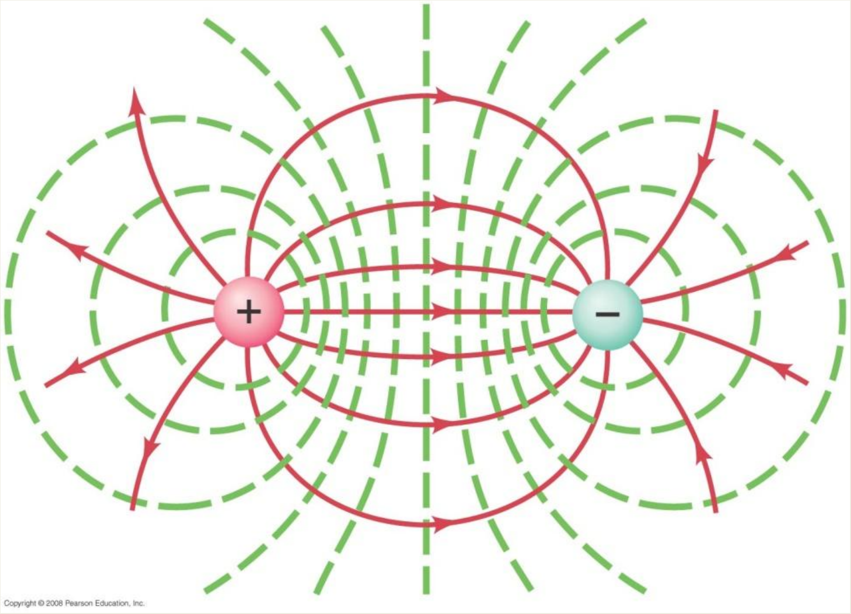


- Lines of equal potential
- Equipotential Surfaces
- Justification for Perpendicularity to the Electric Field
- Mapping of Equipotential Surfaces

Equipotential Lines

- A line in which all points are at the same voltage, is called an *equipotential line*.

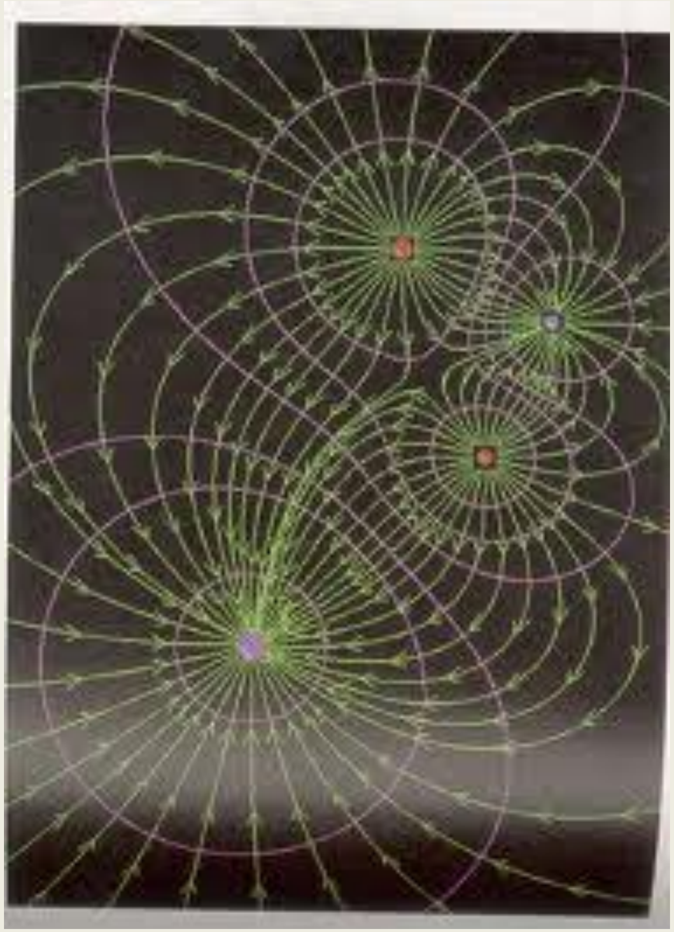
- Equipotential Lines are perpendicular to the electric field



Equipotential Surfaces

- A surface in which all points are at the same voltage is called an *equipotential surface*

- An equipotential surface must be perpendicular to the electric field



Conservative and Non-Conservative Force

A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final positions.

A force is nonconservative when the work it does on a moving object is dependent of the path between the object's initial and final positions.

Conservation of angular momentum

It follows from Newton's second law that:

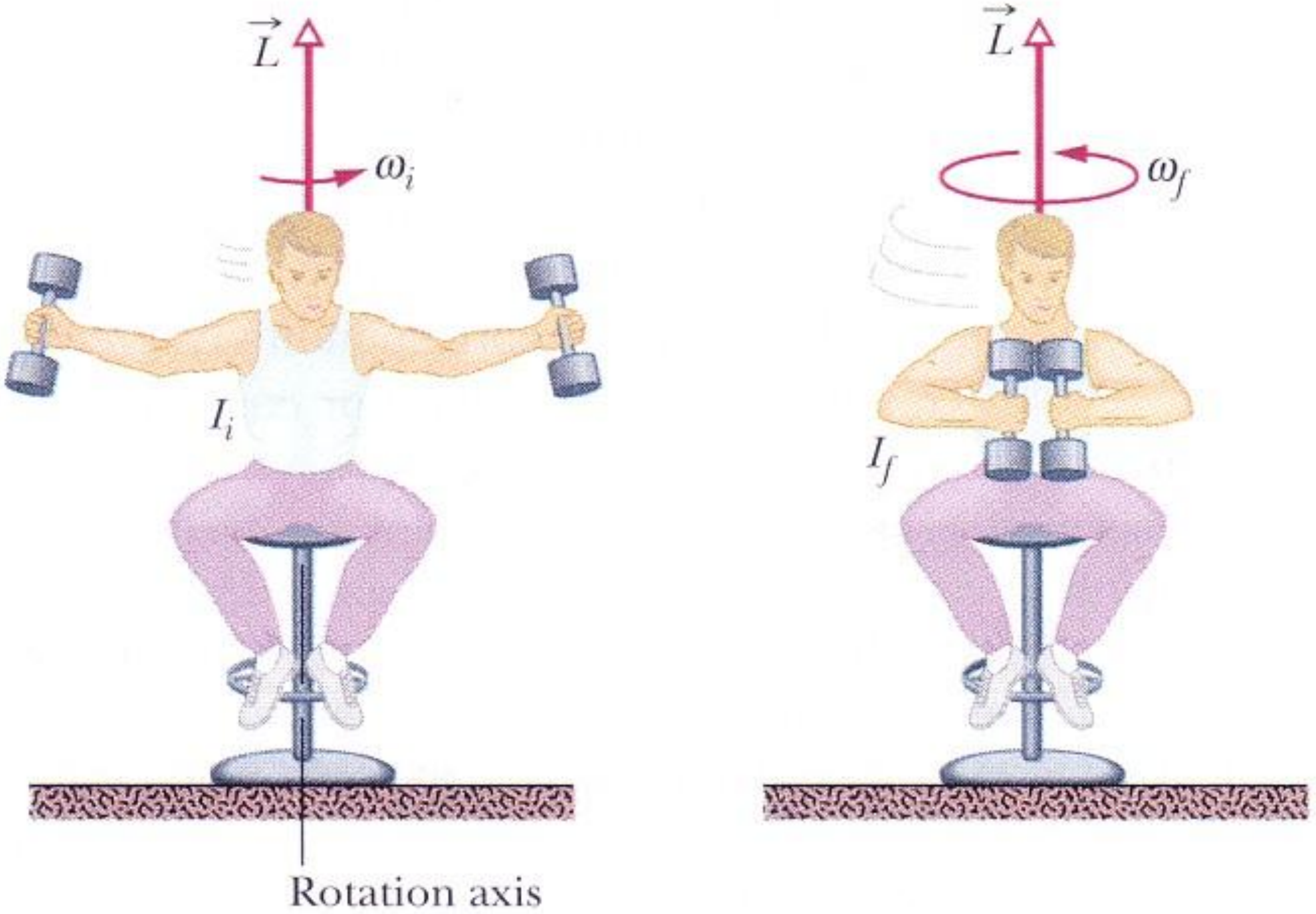
If the net external torque acting on a system is zero, the angular momentum of the system remains constant, no matter what changes take place within the system.

$$\vec{L} = \text{a constant}$$

$$\vec{L}_i = \vec{L}_f$$

$$I_i \omega_i = I_f \omega_f$$

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}$$



Conservation of angular momentum

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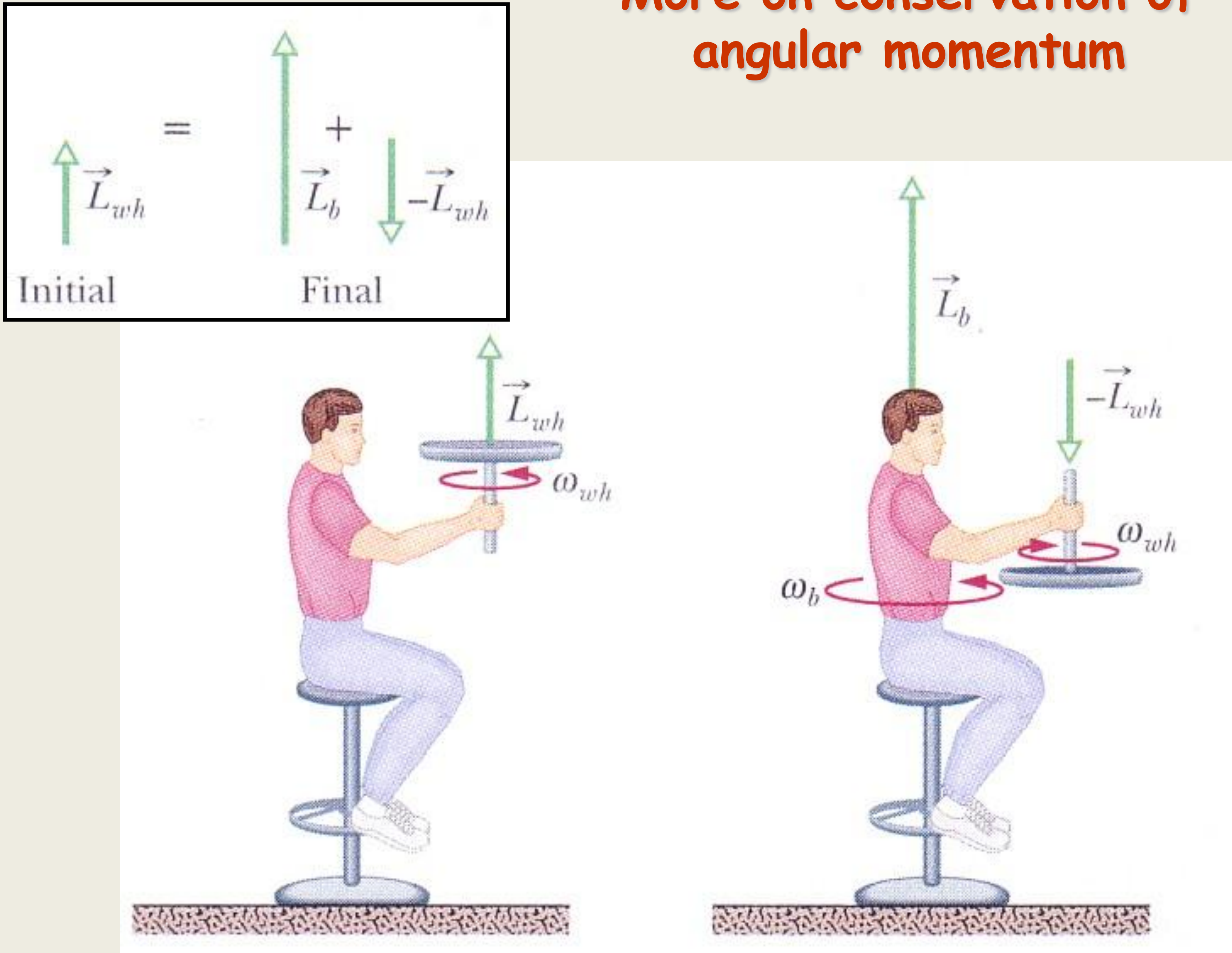
$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}$$

What happens to kinetic energy?

$$K_f = \tfrac{1}{2} I_f \omega_f^2 = \tfrac{1}{2} I_f \left(\frac{I_i^2 \omega_i^2}{I_f^2} \right) = \frac{I_i}{I_f} \tfrac{1}{2} I_i \omega_i^2 = \frac{I_i}{I_f} K_i$$

- Thus, if you increase ω by reducing I, you end up increasing K.
- Therefore, you must be doing some work.
- This is a very unusual form of work that you do when you move mass radially in a rotating frame.
- The frame is accelerating, so Newton's laws do not hold in this frame

More on conservation of angular momentum



- **Inertial Reference Frame:**

- ❑ Any frame in which Newton's Laws are valid
- ❑ Any reference frame moving with uniform motion (***non-accelerated***) with respect to an "absolute" frame "fixed" with respect to the stars.

Perfect Inertial Frame:

The objects/bodies in the universe interact via Gravitational Forces and are present everywhere but are very weak. Hence we can neglect these forces.

Best Approximation: Intergalatic space

- **Non-Inertial Reference Frame:**

- ❑ Is a frame of reference with a changing velocity. The velocity of a frame will change if the frame speeds up, or slows down, or travels in a curved path.
- ❑ is an accelerating frame of reference.
- ❑ is a frame of reference in which Newton's laws of motion do not hold.
- ❑ In a non-inertial frame of reference fictitious forces arise.

Forces in a Rotating Frame

- Let’s take the Earth as the rotating frame. The angular frequency of rotation of the Earth is

$$\omega = \frac{2\pi}{T} = \frac{2\pi\ \text{rad}}{24 \times 3600\ \text{s}} \approx 7.3 \times 10^{-5}\ \text{rad/s}.$$

- We will assume that the inertial frame S and rotating frame *S′*, so the only motion of *S′* relative to S is a rotation with angular velocity ω.
- Newton's laws of motion govern the motion of an object in a (non-accelerating) inertial frame of reference. When Newton's laws are transformed to a uniformly rotating frame of reference, the Coriolis and centrifugal forces appear.

$$\boxed{\vec{F}_{\text{eff}} = \underset{\text{centrifugal}}{-m\vec{\omega}\times(\vec{\omega}\times\vec{r})}\ \underset{\text{coriolis}}{-2m\vec{\omega}\times\vec{v}}}$$

- Both forces are proportional to the mass of the object.
- The Coriolis force is proportional to the rotation rate and the centrifugal force is proportional to its square.
- The Coriolis force acts in a direction perpendicular to the rotation axis and to the velocity of the body in the rotating frame and is proportional to the object's speed in the rotating frame.
- The centrifugal force acts outwards in the radial direction and is proportional to the distance of the body from the axis of the rotating frame.
- These additional forces are termed inertial forces, fictitious forces or pseudo forces.
- They are correction factors that do not exist in a non-accelerating or inertial reference frame.

- The Coriolis effect is a deflection of moving objects when they are viewed in a rotating reference frame.
- the mathematical expression for the Coriolis force was given by French scientist Gaspard-Gustave Coriolis.

$$\boxed{\vec{F}_{\text{eff}} = \underbrace{-m\vec{\omega}\times(\vec{\omega}\times\vec{r})}_{\text{centrifugal}} \underbrace{-2m\vec{\omega}\times\vec{v}}_{\text{coriolis}}}$$

Effect of Centrifugal force on g

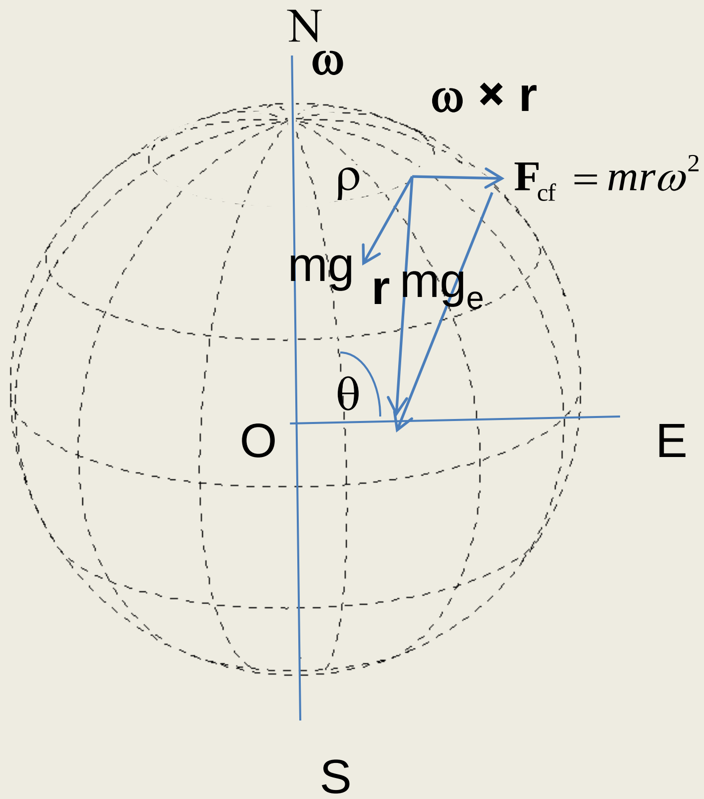
$$g_e = g - R\omega^2 \cos^2 \theta$$

$$\text{At poles}, (\theta = 90°)\ g_e = g$$

$$\text{At equator}, (\theta = 0°)\ g_e = g - R\omega^2$$

Non–Vertical Gravity

Due to the centrifugal force, a plumb bob does not actually point in the direction to the center of the Earth except at the pole or equator.



Coriolis Force

$$\vec{F}_{cor} = -2m(\vec{\omega} \times \vec{v})$$

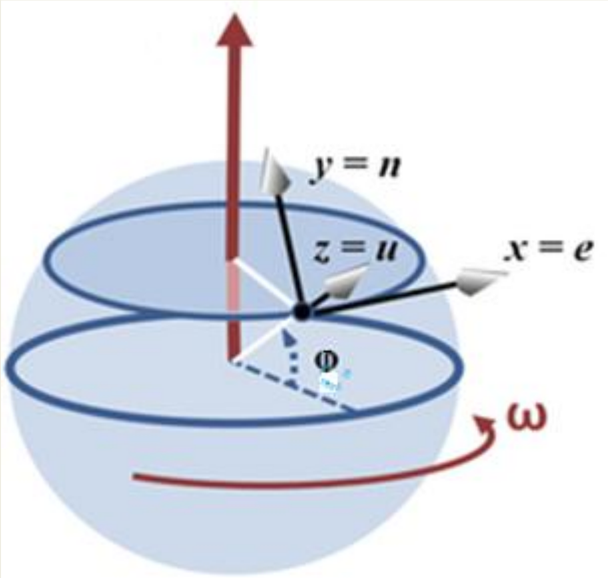
$$\text{if } \vec{\omega} = \omega\cos\theta\hat{j} + \omega\sin\theta\hat{k} \quad \text{and } \vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\text{then } \vec{F}_{cor} = \vec{F}_h + \vec{F}_v$$

$$= (2m\omega v_y \sin\theta\hat{i} - 2m\omega v_x \sin\theta\hat{j}) + (2m\omega v_x \cos\theta\hat{k})$$

$$\left|\vec{F}_h\right| = 2m\omega v\sin\theta \quad \because (\mathrm{v} = \sqrt{v_x^2 + v_y^2})$$

$$\left|\vec{F}_v\right| = 2m\omega v_x \cos\theta$$



Coordinate system at latitude θ with x-axis east, y-axis north and z-axis upward (radially outward from center of sphere).

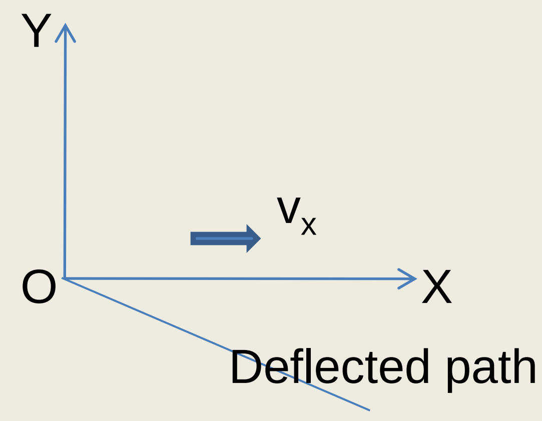
- Main effect will be due to horizontal component of F_{cor}
- Vertical comp. is along +ve z-direction, i.e. it acts in a direction opposite to acceleration due to gravity. So its effect will be neglected in most cases.

$$\vec{F}_{cor} = 2m\omega v_y \sin\theta\hat{i} - 2m\omega v_x \sin\theta\hat{j}$$

if particle is moving along x-axis,

$$v_y = 0,\ \vec{F}_{cor} = -2m\omega v_x \sin\theta\hat{j}$$

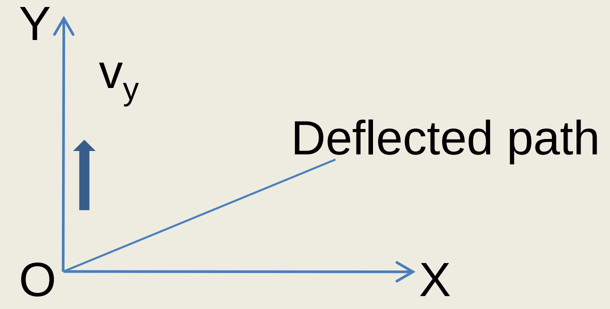
i.e., deflection is towards -ve y-axis.



if particle is moving along y-axis,

$$v_x = 0,\ \vec{F}_{cor} = 2m\omega v_y \sin\theta\hat{i}$$

i.e., deflection is towards +ve x-axis.



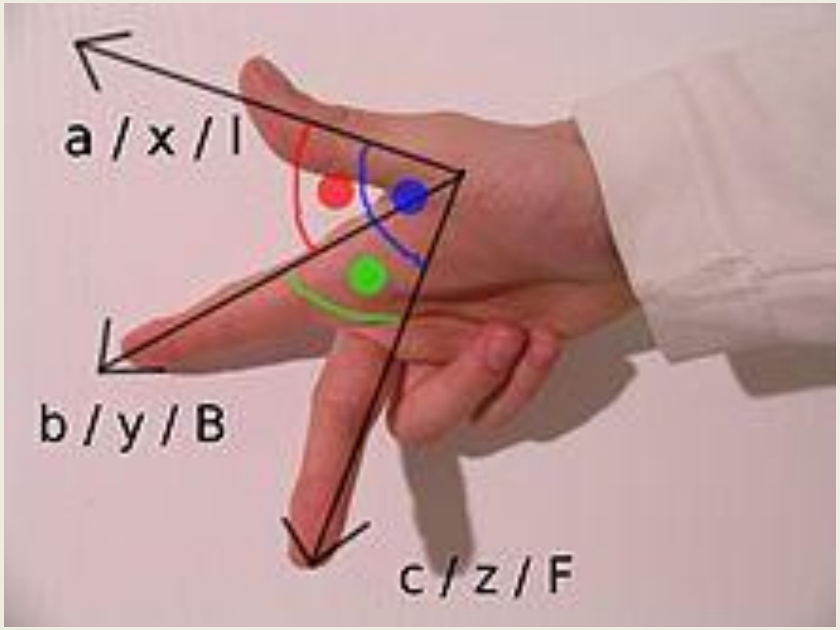
- i.e., particle will always be deflected towards its right in N-hemisphere.

RIGHT-HAND RULE

- two vectors *a* and *b* that has a result which is a vector *c* perpendicular to both *a* and *b*.

$$\vec{a} \times \vec{b} = \vec{c}$$

- With the thumb, index, and middle fingers at right angles to each other (with the index finger pointed straight), the middle finger points in the direction of *c* when the thumb represents *a* and the index finger represents *b*.

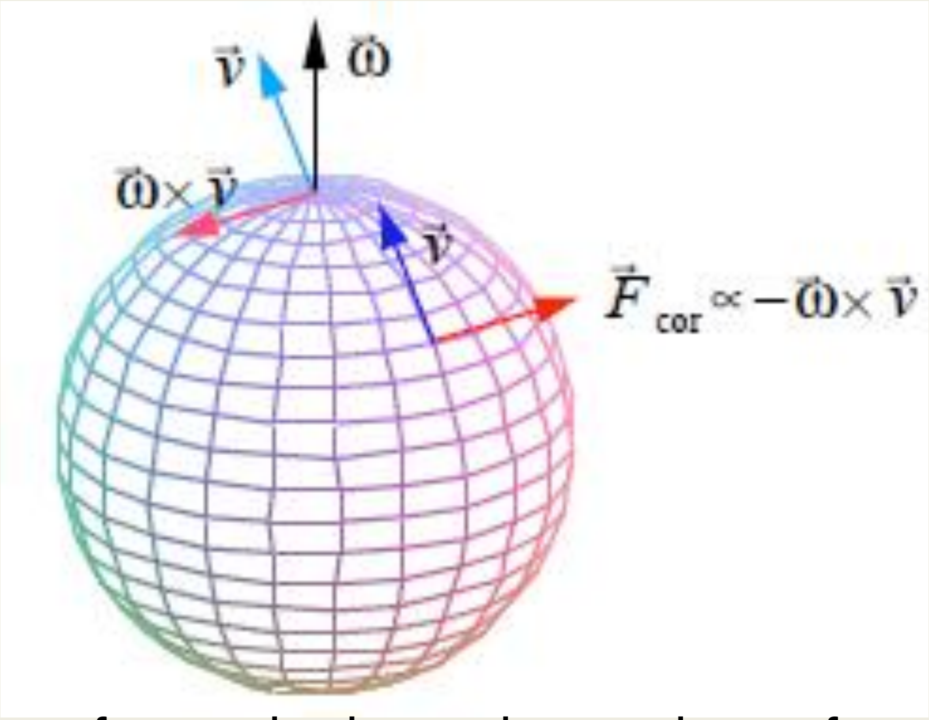


Direction of the Coriolis Force

Case 1: in N-hemisphere

If body is moving towards NORTH

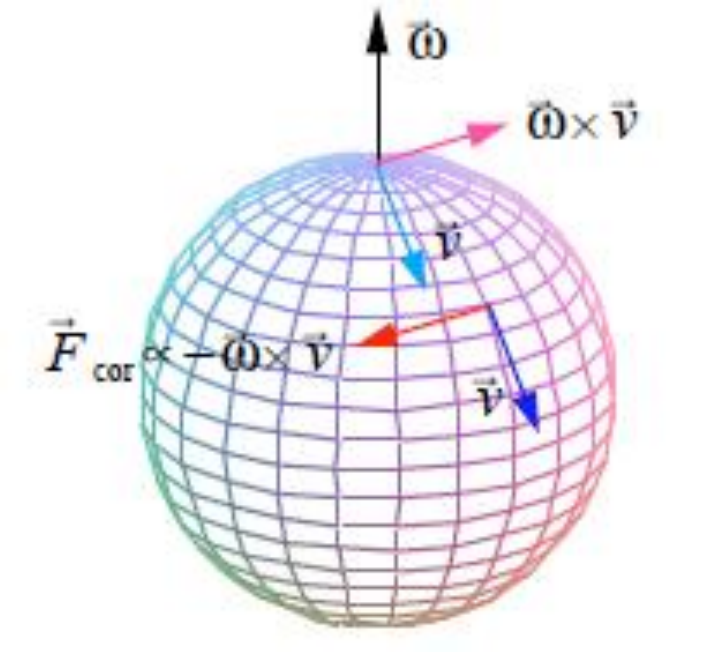
Coriolis force: EASTWARD



Case 2: in N-hemisphere

If body is moving towards SOUTH

Coriolis force: WESTWARD



- for any body moving on the surface of the earth in the NORTHERN-hemisphere, the coriolis force deflects it to the *RIGHT*
- for any body moving on the surface of the earth in the SOUTHERN-hemisphere, the coriolis force deflects it to the *LEFT*

Effect of Coriolis Force on Freely Falling objects

- Consider an object at the surface of the Earth in free-fall with no other forces acting (i.e. no air resistance), then deflection due to coriolis force is

$$x = \frac{\omega g t^3 \cos\theta}{3} = \frac{\omega \cos\theta}{3}\left(\frac{8h^3}{g}\right)^{1/3}$$

- And coriolis acceleration is

$$\vec{a} = 2\omega g t \cos\theta \hat{i}$$

- And therefore direction of deflection will also be along east and deflection is independent of mass of the object.

Coriolis Effect

- For air travelling northwards in the northern hemisphere, there is an eastward acceleration
- For air travelling southwards in the northern hemisphere, there is a westward acceleration

  (always deflected towards RIGHT)

- For air travelling northwards in the southern hemisphere, there is an westward acceleration
- For air travelling southwards in the southern hemisphere, there is a eastward acceleration

  (always deflected towards LEFT)

Coriolis Effect

- If air moves in east-west direction
  - If it moves eastward, it would take less time to complete one entire rotation as it is travelling faster than the earth's surface
  - If it moves westward, it is opposing the earth's rotation and would take longer to complete a rotation

Geographical effects of Coriolis Force

- The Coriolis effect is caused by the rotation of the Earth and the inertia of the mass experiencing the effect.
- Because the Earth completes only one rotation per day, the Coriolis force is quite small, and its effects generally become noticeable only for motions occurring over large distances and long periods of time, such as large-scale movement of air in the atmosphere or water in the ocean.

Foucault Pendulum

- The **Foucault's pendulum**, named after the French physicist Léon Foucault, is a simple device conceived as an experiment to demonstrate the rotation of the Earth.

- While it had long been known that the Earth rotated, the introduction of the Foucault pendulum in 1851 was the first simple proof of the rotation in an easy-to-see experiment.

Apparatus Details

- The first public exhibition of a Foucault pendulum took place in February 1851 in Paris Observatory.

- Foucault suspended a 28 kg brass-coated lead bob with a 67 meter long wire. The period of oscillation was nearly 17 seconds.

- Under the pendulum was a sand hillock where the pendulum leave its changing traces. Thus people could easily recognize the rotational movement of the Earth.

- The large length of the pendulum increases its period of oscillation

$$T = 2\pi\sqrt{l / g}$$

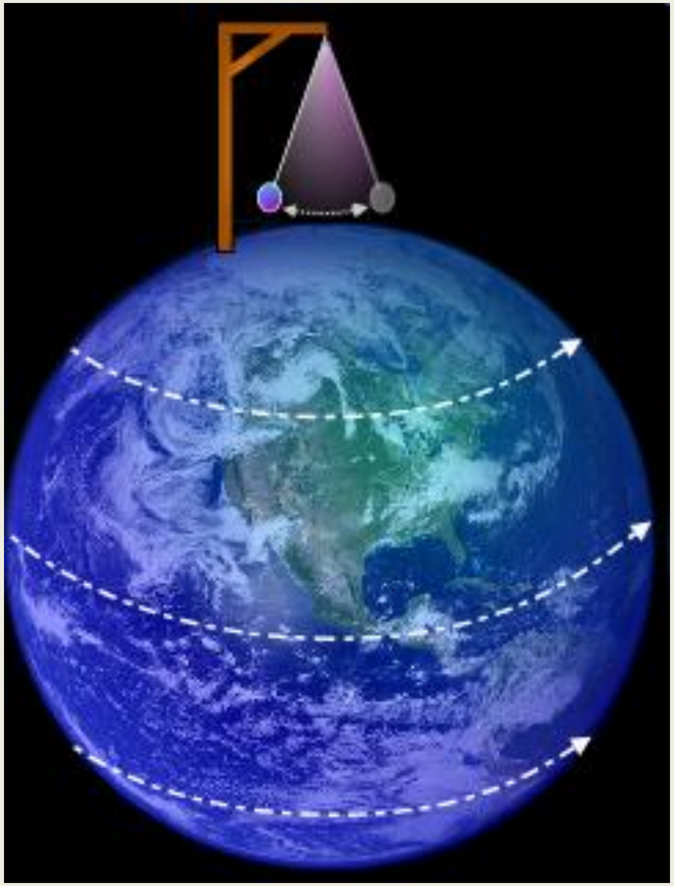
Pendulum at Poles

- Suppose someone put a pendulum above the South Pole and sets it swinging in a simple arc.

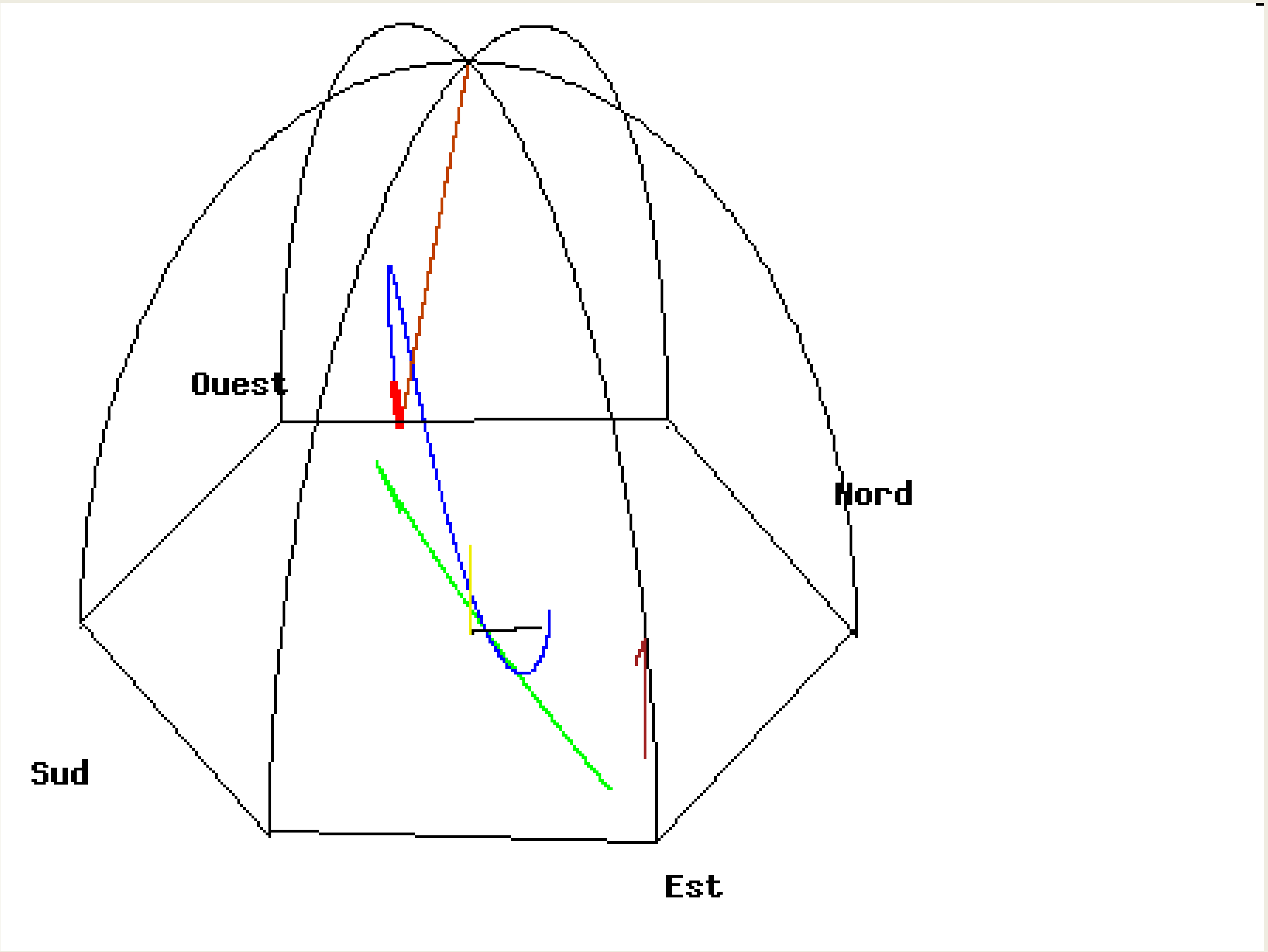
- To someone directly above the Pole and not turning with the earth, the pendulum would seem to trace repeatedly an arc in the same plane while the earth rotated slowly below it.

- To someone on the earth, however, the earth seems to be stationary, and the plane of the pendulum's motion would seem to move slowly, viewed from above.

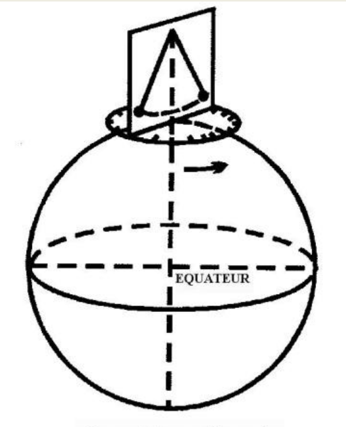
- We say that the pendulum's motion *precesses*. The earth turns on its axis every 23.93 hours, so to the terrestrial observer at the pole, the plane of the pendulum seems to precess through 360 degrees in that time.



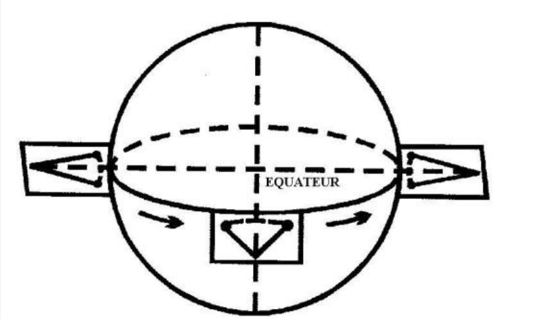
- http://en.wikipedia.org/wiki/Foucault_pendulum
- http://www.animations.physics.unsw.edu.au/jw/foucault_pendulum.html
- http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/FoucaultSimple/FoucaultSimple.html



Direction of rotation of plane of oscillation



Le pendule au pôle nord



Le pendule à l'équateur

- For observers at the Northern hemisphere, the direction of pendulum rotation is clockwise
- and for observers at the Southern hemisphere, rotation is opposite in direction
- The observed rotational period of the pendulum (rotation angle of 360°) depends on the latitude

Forces acting on pendulum

- weight mg (acting towards center of earth)
- Tension T in the string
- coriolis force
- centrifugal force

$$\vec{F} = m\vec{g} + \vec{T} - 2m(\vec{\omega} \times \vec{v}) - m\vec{\omega} \times (\vec{\omega} \times r)$$

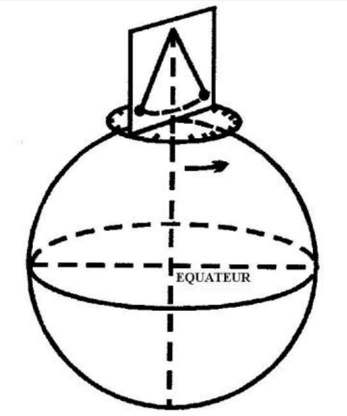
due to smaller angular ve locty of earth,

fourth tem in the equation can be neglected

$$\therefore \vec{F} = m\vec{g} + \vec{T} - 2m(\vec{\omega} \times \vec{v})$$

- We also consider amplitude of oscillations small enough so that motion of pendulum bob is confined to xy-plane.

Period of rotation of the plane of oscillation

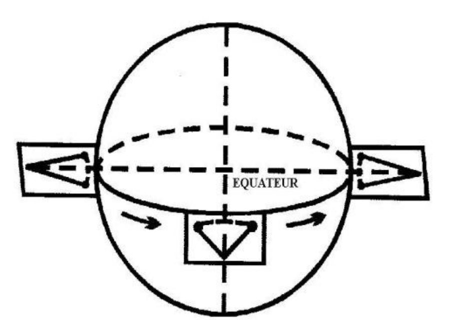


Le pendule au pôle nord

$$T_{rot} = \frac{2\pi}{\omega \sin \theta} = \frac{24\,hrs}{\sin(\theta)}$$

$$\text{where } \theta = \text{lattitude}$$

$$\text{and } 2\pi/\omega = 24\,hrs$$



Le pendule à l'équateur

The pendulum located at the poles (θ=90 degree) has a rotational period of 24 h, while at the equator (θ=0 degree) the effect of rotation is not observed.

Simple Harmonic Motion

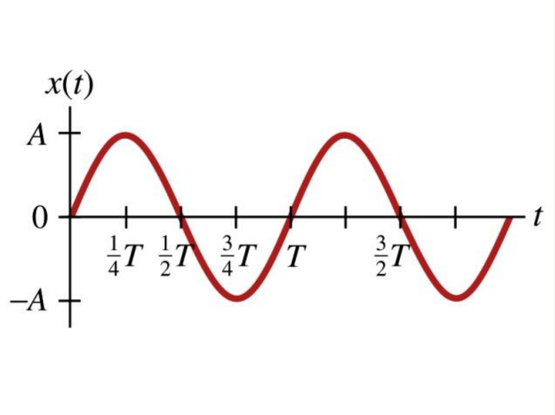
- Other variables frequently used to describe simple harmonic motion:

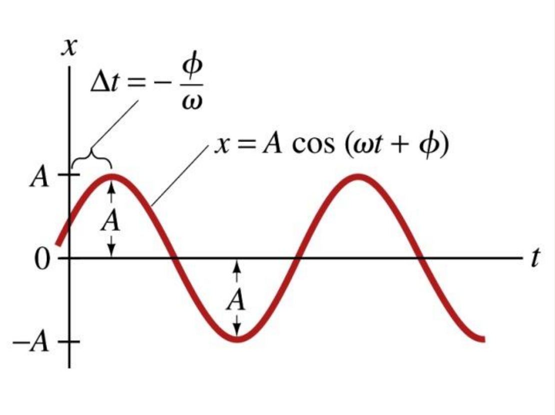
  – The period T: the time required to complete one oscillation. The period T is equal to $2\pi/\omega$.

  – The frequency of the oscillation is the number of oscillations carried out per second:

    $\nu = 1/T$

    The unit of frequency is the Hertz (Hz). Per definition, 1 Hz = 1 s^{-1}.





Simple Harmonic Motion
What forces are required?

- Using Newton's second law we can determine the force responsible for the harmonic motion:

$$F = ma = -m\omega^2 x$$

- The total mechanical energy of a system carrying out simple harmonic motion is constant.

- A good example of a force that produces simple harmonic motion is the spring force: $F = -kx$. The angular frequency depends on both the spring constant k and the mass m:

$$\omega = \sqrt{(k/m)}$$

Simple Harmonic Motion (SHM). The equation of motion.

- All examples of SHM were derived from he following equation of motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$x(t) = A\cos(\omega t + \alpha) + B\sin(\omega t + \beta)$$

- The most general solution to the equation is

Simple Harmonic Motion (SHM). The equation of motion.

- If $A = B$

$$x(t) = A\cos(\omega t + \alpha) + B\sin(\omega t + \beta) =$$

$$= A\left(\sin\left(\frac{1}{2}\pi - \omega t - \alpha\right) + \sin(\omega t + \beta)\right) =$$

$$= 2A\sin\left(\frac{1}{4}\pi + \frac{\beta}{2} - \frac{\alpha}{2}\right)\cos\left(\frac{1}{4}\pi - \omega t - \frac{\beta}{2} - \frac{\alpha}{2}\right)$$

which is SHM.

Damped Harmonic Motion.

- Consider what happens when in addition to the restoring force a damping force (such as the drag force) is acting on the system:

$$F = -kx - b\frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

- The equation of motion is now given by:

Damped Harmonic Motion.

- The general solution of this equation of motion is

$$x(t) = Ae^{i\omega t}$$

- If we substitute this solution in the equation of motion we find

$$-\omega^2 Ae^{i\omega t} + i\omega \frac{b}{m} Ae^{i\omega t} + \frac{k}{m} Ae^{i\omega t} = 0$$

- In order to satisfy the equation of motion, the angular frequency must satisfy the following condition:

$$\omega^2 - i\omega \frac{b}{m} - \frac{k}{m} = 0$$

Damped Harmonic Motion

- We can solve this equation and determine the two possible values of the angular velocity:

$$\omega = \frac{1}{2}\left(i\frac{b}{m} \pm \sqrt{4\frac{k}{m} - \frac{b^2}{m^2}} \right);\ \frac{1}{2} i\frac{b}{m} \pm \sqrt{\frac{k}{m}}$$

- The solution to the equation of motion is thus given by

$$x(t);\ x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}}$$

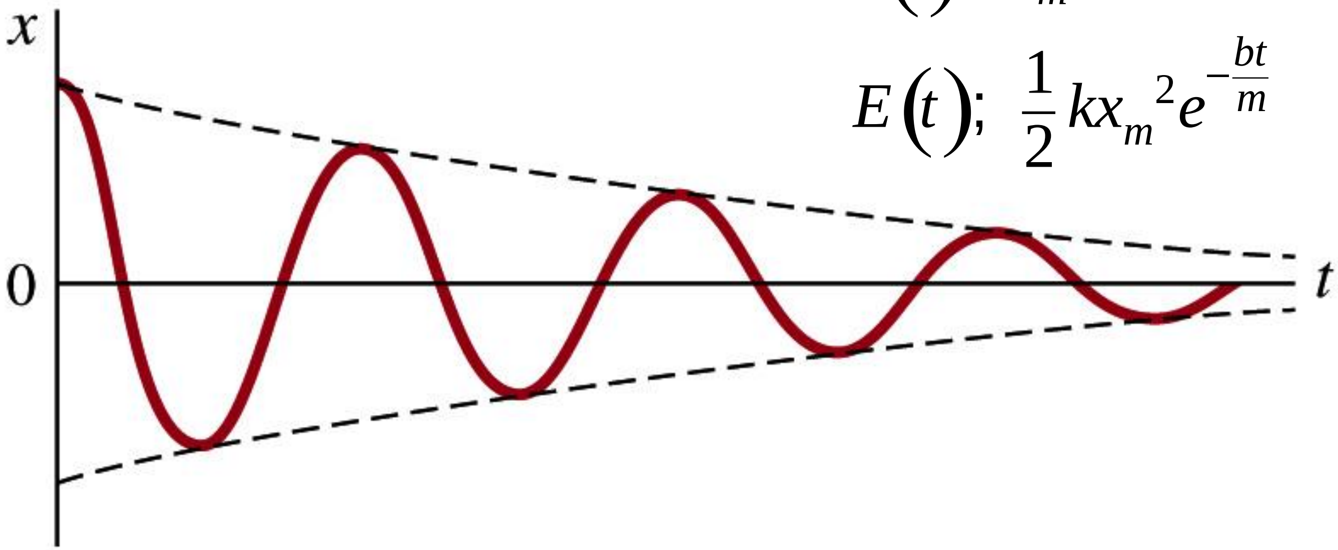
SHM Term

Damping Term

Damped Harmonic Motion.

$$x(t);\ x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}}$$

$$E(t);\ \frac{1}{2} k x_m{}^2 e^{-\frac{bt}{m}}$$



The general solution contains a SHM term,
with an amplitude that decreases as function of time

Driven Harmonic Motion.

- Consider what happens when we apply a time-dependent force $F(t)$ to a system that normally would carry out SHM with an angular frequency ω_0.

- Assume the external force $F(\mathrm{t}) = mF_0\sin(\omega t)$. The equation of motion can now be written as

$$\frac{d^2x}{dt^2} = -\omega_0{}^2 x + F_0 \sin\left(\omega t\right)$$

- The steady state motion of this system will be harmonic motion with an angular frequency equal to the angular frequency of the driving force.

Driven Harmonic Motion.

- Consider the general solution

$$x(t) = A\cos(\omega t + \phi)$$

- The parameters in this solution must be chosen such that the equation of motion is satisfied. This requires that

$$-\omega^2 A\cos(\omega t + \phi) + \omega_0{}^2 A\cos(\omega t + \phi) - F_0 \sin(\omega t) = 0$$

- This equation can be rewritten as

$$\left(\omega_0{}^2 - \omega^2\right) A\cos(\omega t)\cos(\phi) -$$
$$\left(\omega_0{}^2 - \omega^2\right) A\sin(\omega t)\sin(\phi) - F_0 \sin(\omega t) = 0$$

Driven Harmonic Motion.

- Our general solution must thus satisfy the following condition:

$$\left(\omega_0{}^2-\omega^2\right)A\cos\left(\omega t\right)\cos\left(\phi\right)-\left\{\left(\omega_0{}^2-\omega^2\right)A\sin\left(\phi\right)-F_0\right\}\sin\left(\omega t\right)=0$$

- Since this equation must be satisfied at all time, we must require that the coefficients of cos(ωt) and sin(ωt) are 0. This requires that

$$\left(\omega_0{}^2-\omega^2\right)A\cos\left(\phi\right)=0$$

and

$$\left(\omega_0{}^2-\omega^2\right)A\sin\left(\phi\right)-F_0=0$$

Driven Harmonic Motion.

- The interesting solutions are solutions where $A \neq 0$ and $\omega \neq \omega_0$. In this case, our general solution can only satisfy the equation of motion if

$$\cos(\phi) = 0$$

and

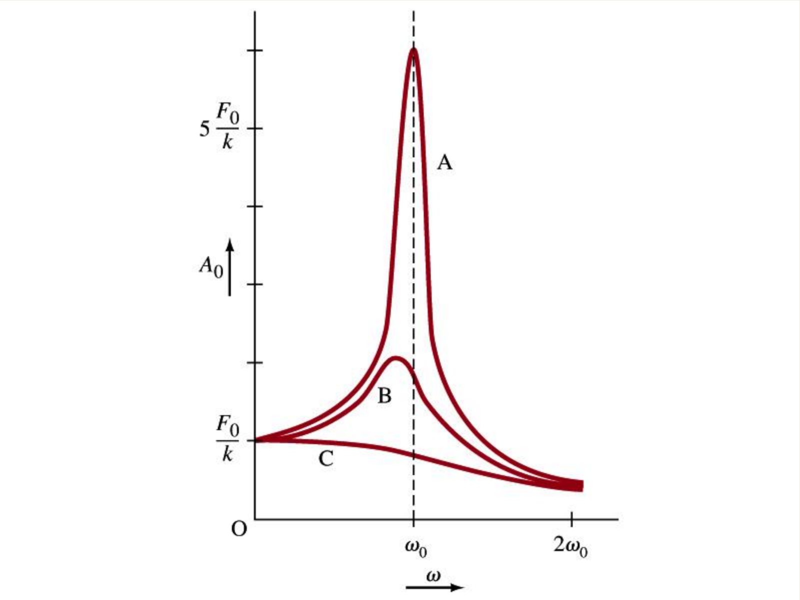
$$\left(\omega_0{}^2 - \omega^2\right) A \sin(\phi) - F_0 = \left(\omega_0{}^2 - \omega^2\right) A - F_0 = 0$$

- The amplitude of the motion is thus equal to

$$A = \frac{F_0}{\left(\omega_0{}^2 - \omega^2\right)}$$

Driven Harmonic Motion.

- If the driving force has a frequency close to the natural frequency of the system, the resulting amplitudes can be very large even for small driving amplitudes. The system is said to be in resonance.
- In realistic systems, there will also be a damping force. Whether or not resonance behavior will be observed will depend on the strength of the damping term.



Future Scope and relevance to industry

Research topics:

- https://www.researchgate.net/publication/228876236_An_introduction_to_electrical_resistivity_in_geophysics/figures?lo=1&utm_source=google&utm_medium=organic
- https://www.researchgate.net/publication/313147105_Beyond_the_Point_Charge_Equipotential_Surfaces_and_Electric_Fields_of_Various_Charge_Configurations

NPTEL/other online link

- https://nptel.ac.in/courses/117103065/12
- https://nptel.ac.in/courses/117103065/modules/chap2/slide/slide27.htm
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- https://www.youtube.com/watch?v=AXDseVTQkbc
- https://nptel.ac.in/courses/115106090/20